Strategic Audit Policies Without Commitment

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Summary. This chapter constructs and analyzes a simple auditing model in order to answer questions concerning three principal issues: (i) the information contained in the report, (ii) commitment to the audit policy and (iii) audit effort. The approach taken is based on the concept of perfect Bayesian equilibrium. We attempt to examine the nature of such equilibria and arguments as to which equilibrium one would expect to observe.

Key words: audit policies, audit game, commitment, perfect Bayesian equilibrium

1 Introduction

This paper constructs and analyzes a simple model of auditing in which three principal issues are explored, namely: (i) THE INFORMATION CONTAINED IN THE REPORT. An audit is a process of verification of a report of private information available to the reporter but not to the auditor. What information is contained in a report? Is it sufficient for the auditor to infer the private information exactly or is it imperfect? How does this affect what the auditor does? (ii) COMMITMENT TO THE AUDIT POLICY-HOW DOES ITS ABSENCE AFFECT REPORTING AND INVESTIGATION DECISIONS. Can the auditor commit in advance to an audit policy, even when it may not be optimal to carry out the policy at the time of implementation, and, related to this, does auditing have only a purely deterrent role or can it lead to recovery of assets as well? (iii) AUDIT EFFORT. How is audit intensity or effort determined?

Our attempts to answer these questions will involve using the concept of Perfect Bayesian Equilibrium; we will attempt to examine the nature of such equilibria and arguments as to which equilibrium one would expect to observe.

* Arijit Mukherji tragically passed away in October 2000. This paper is dedicated to his memory.

The paper therefore also serves the purpose of introducing the ideas of equilibrium refinements (and the effects of assuming a player can commit to a sequence of actions in advance) to the audience for this book and illustrating the usefulness of these refinements through an important application. The partially expository nature of our objectives mean that we have explained proofs and examples in more detail than we would normally have chosen to do.

We present the basic structure of our model in an informal fashion in this introduction, and compare it to earlier work. In the next section the model is specified, emphasizing the nature of the ability to commit or its absence. We formally derive results for the case when auditing results in perfect discovery in Sections 3 and 4. In order to examine audit effort or intensity Section 5 assumes that an audit is imperfect so that repeated auditing may be necessary to verify the report. We conclude by discussing potential empirical implications of the competing theories of auditing. All proofs are contained in an appendix.

The game that we consider has two players—a manager, who observes the true value of the firm he or she manages and who decides whether to consume some part of this value as perquisites, and an auditor, who does not know the firm's true value but is retained by the firm's shareholders to monitor the manager's report of the value. The difference between the true and the reported value then constitutes the unauthorized consumption of perquisites by the manager. To focus on the interaction between the audit and reporting strategy, we assume the auditor has no moral hazard problem in auditing and acts in the interests of shareholders.

In our model, "nature" moves first and draws a value for the firm from a commonly known probability distribution. The manager observes this value and decides how much to report, retaining the residual as a perquisite. The auditor observes the report and decides whether or not to audit at a cost c. As an important distinguishing feature of our model, we assume that the auditor cannot commit to the audit strategy in advance of the report, but must use the information conveyed by the report in the audit decision. Assume for the moment that an audit consists of a single observation that leads to perfect discovery (This is relaxed later). If there is no audit, the manager obtains a payoff corresponding to the difference between true and reported value and the auditor (shareholders) obtains the reported amount.[3] If there is an audit, the auditor obtains the whole value of the firm less the cost of observation and the

[3] We assume that both the manager and the auditor are rational economic agents. In a tax audit context, Erard and Feinstein [EF94] consider the implications of assuming that some taxpayers are intrinsically honest and will not misreport their true taxable income. In a model of analytical review, Newman et al. [NPS99] consider a model in which the auditee is honest with some probability, and fraudulent with the complementary probability. In our model, if the manager were able to consume the residual, undetected, then he would prefer to do so. Graetz et al. [GRW86] were the first to consider intrinsically honest taxpayers.

manager must pay a penalty proportional to the amount of underreporting. This penalty could be thought of as being nonpecuniary in nature, and hence not accruing to any individual.

We characterize a large number of equilibria in this model, including some which resemble the audit policy obtained if the auditor *can* commit in advance to the audit policy. This multiplicity of equilibria results from the many different interpretations that the auditor can place on a report, corresponding to different reporting strategies, all of which provide the same information to the auditor about the value of an audit. The equilibria of this model all involve some pooling—managers with different true values make the same report. Therefore auditing potentially has an information acquisition role in any pooling equilibrium. The report identifies, in equilibrium, a range of values which the manager may have observed. If the report is audited, the actual value is discovered in these cases, therefore producing information that was unavailable before the audit. Some equilibria also involve partially separating reporting strategies—manager with different values make different reports. In the range of values in which the equilibrium is separating, the true value can be inferred exactly from the report and the role of the audit is purely to deter. However, the auditor would still want to audit, since recovery of the fraud amount involves verification that stealing has actually occurred.

Among all these equilibria, we show how to choose one as most plausible. The equilibrium that we will argue for involves pooling only at the lower end of the range of values and separation at all values above a cutoff. This maximally separating—and therefore maximally informative—equilibrium is chosen by using the D1 refinement of sequential equilibrium proposed by Banks and Sobel [BS87]. A pooling reporting strategy will have many out-of-equilibrium reports that should never be sent by the manager, and D1 places restrictions on what interpretation the auditor can make if such reports are received. These interpretations must be credible because it is the auditor's response to those reports that ensure that they are never sent. In our context, what matters is the strong monotonicity property this refinement associates with beliefs. For any "unexpected move" (deviation from equilibrium), D1 requires that the auditor believe that the manager observed that value of the firm that would make such a deviation most desirable. This will rule out all but the maximally informative equilibrium.[4]

The potential empirical implications of our analysis use the maximally separating equilibrium as the basic prediction. We compare the qualitative features of this equilibrium with those when the auditor can commit. In the maximally separating equilibrium, every type of manager understates the value of the firm. Audit probabilities are responsive and strictly decreasing in the report. The prior distribution of firm values affects the equilibrium only towards the two ends of the support. In contrast, the commitment equilibria have audit probabilities that are constant over a lower range of reports. The manager

[4] Reinganum [Rei93] uses a similar device in a different context of plea bargaining.

understates the report only when it will never be audited. The prior distribution plays a crucial role in these other equilibria by changing the intervals of reports that characterize the equilibrium.

We now consider how this model helps us to pose the questions we are interested in exploring. Like Fellingham and Newman [FN85], an auditor and a manager choose their strategies optimally given the conjectures each has of the other's behavior (In other words, the problem is formulated as an explicit extensive form game and therefore amenable to equilibrium analysis). In Fellingham and Newman's version, the manager has no private information and the auditor and manager move simultaneously, one choosing whether to commit fraud and the other whether to audit. Their framework does not allow auditing to have any informational role, only one of pure deterrence. It is clear that adding a reporting stage to their game without introducing private information will not be enough to induce any qualitatively different conclusions, since both the manager who has committed fraud and the one who has not will find it optimal to deny fraud. In our approach, on the other hand, the potential informational role of auditing, in an environment where strategic misreporting could occur, can be examined along with its deterrent aspect. This leads to a richer and more complete strategic analysis.

The second major issue is that of commitment. In the tax audit literature especially, models have been proposed with features similar to ours except that the order of moves between auditor and manager (or taxpayer) is reversed. In these papers (Morton [Mor93], Sanchez and Sobel [SS93], Border and Sobel [BS87], Reinganum and Wilde [RW85], for example) the auditor announces a policy to which he or she is committed no matter what information is conveyed by the manager's report. The equilibrium in such a model consists of the auditor auditing every report below a certain cutoff with the same constant probability and auditing reports above the cutoff with zero probability. The manager (or taxpayer) reports the value truthfully up to the cutoff. If the value of the firm is above the cutoff, the manager reports the cutoff value. Thus only those who, in equilibrium, do not commit fraud are audited. The auditor expects not to find any underreporting when he or she audits, though she is committed to incur the costs of such an audit. There are several means by which such a commitment can be sustained, such as bonding, reputation effects or delegation. We will discuss these below, but each appears to be somewhat problematic. Our model offers an exploration of the policies that may result in the absence of such commitment and an elucidation of the distinctions between the two approaches.

The paper most similar to ours is Reinganum and Wilde [RW86], (especially the appendix), who analyze a similar reporting game in a tax audit context. The differences between their analysis and ours are as follows:

1. They consider only a single, perfectly informative equilibrium, whereas we find all the pure reporting strategy equilibria and show how to refine these to a unique equilibrium.

2. Their model allows for an unbounded amount of fraud, by assuming that there is a negative income tax. Since the manager cannot steal more than the value of the firm, we place a lower bound on the amount of fraud. We show this will rule out all perfectly informative equilibria.
3. We also directly model the audit technology, based on the nature of audit sample information, and consider two distinct ways of modelling audit intensity. (While the appendix of Reinganum-Wilde deals with the single audit case we consider in the text, the main body of their paper can be interpreted as an analysis of audit intensity, though this is not linked to the single audit case as is done here. They refer to the single-audit case as the costs being linear with respect to probability.)

We should say, however, that we acknowledge that Reinganum and Wilde [RW86] was the first paper to raise the commitment issue and to analyse the consequences of no commitment. Theirs is clearly the pioneering paper in this area, though we feel that we too have made a contribution as described above.

Another recent paper, Khalil [Kha97], has a title very similar to ours, though the model he discusses is somewhat different. His paper is in the context of regulation, modelled as a principal-agent problem with monitoring. The principal first proposes a contract, the agent who could be one of two types either accepts or rejects the contract, and if she accepts produces a level of output. Given the output, the principal could choose to audit or not, to determine if in fact the agent has produced the contractual output corresponding to his type. Our model is with a continuum of types and we do not have a contracting or production stage. The paper does, of course, address similar issues of commitment and incentives to audit.

The third major area, imperfect audits and audit effort, is analyzed in section 5. When one unit of audit cost will discover a misstatement only probabilistically, the audit may be repeated in order to gain higher confidence in the report. This results in an equilibrium intensity of auditing and a model of audit effort in which the auditor does not obtain perfect assurance in the manager's report. Baiman et al. [**?**] model a three agent contracting problem between the owner and manager of a firm and an independent auditor. Although contracting issues among these agents are of high interest, their results seem too strong, since they show that the auditor will always be motivated to choose effective auditing to obtain full information whenever he is engaged. This prevents the auditor from using a strategy which is contingent on the manager's report as well as partial auditing to obtain less than full information. Our approach is to make exogenous but plausible assumptions about the contracting relationship in order to focus on the details of the audit and reporting strategy.

In a previous version of this paper, we showed that our framework applies also to reporting value to the financial markets. The risk in this case is that the manager's report will be overstated (e.g., higher income or assets than is permitted by accounting principles) so that the manager can show better performance in order to obtain bonuses or promotion. We argue that the risks and benefits to both the auditor and manager for such misstatements are qualitatively the same as for asset fraud. There is a perfectly informative equilibrium for reporting fraud. The other main difference is that the audit probability schedule is now an increasing function of the manager's report. In the interests of space, this extension does not appear in the paper.

2 The Perfect Audit Game

This section describes the benchmark case of asset fraud and perfect auditing. Imperfect auditing and reporting fraud will be considered later. The game has two players, a manager of a firm and an auditor. As an insider, only the manager knows the value, v, of the firm, so let v be a random variable with a continuous probability density function $f(\cdot)$, on bounded support $[0, V]$. The manager must issue a report, r, on the value of the firm. By underreporting the value, $r < v$, the manager can obtain rents from the firm in which the residual $v - r$ is appropriated to his own use, i.e., asset fraud. The amount the manager can report is restricted to lie in the interval $[0, v]$: the manager will not contribute to the firm from his own pocket and cannot take more than the value of the firm for his own use.

The auditor observes the report and may then choose to conduct an audit at a cost $c > 0$, which will perfectly reveal v and the amount of the fraud. If an audit reveals a misreport, then the manager must return the amount of the fraud and will suffer some penalty which is assumed to be in proportion to the amount of the fraud, $M(v - r)$, with $M > 0$. Acting in the interests of the owner, the auditor wishes to minimize the expected amount of misreporting net of audit cost. Formally, the expected payoffs to the manager and expected costs to the owner, respectively, when the manager observes v, reports r and the auditor audits with probability p, are

$$U = (1 - p)(v - r) - pM(v - r) = [1 - p(M + 1)](v - r)$$

$$C = pc + (1 - p)(v - r) = (v - r) + p[r - (v - c)].$$

The basic incentives in this game are straightforward to describe. The manager wishes to report as little as possible, except to the extent that the audit deters him. In particular, the manager will be attracted to low reports which are never audited and carry no risk of discovery. Further, if a report is always audited or, in fact, audited with any probability greater than $1/(1 + M)$, (the probability which makes the manager's expected payoffs identically zero) the manager will never choose that report unless he is being truthful. As for the auditor, the manager's report may convey some information about the value of the firm, so the auditor may wish to use this report to calculate the expected value of the firm. A costly audit will be undertaken only when all available information suggests that a sufficient amount of misreporting will be discovered to justify the audit cost.

The equilibrium concept that formally captures this is the sequential equilibrium of Kreps and Wilson [KW82] (This paper does not define the equilibrium concept for infinite strategy spaces, so effectively we use the Perfect Bayesian Equilibrium concept of Fudenberg and Tirole – see their textbook [FT91] for an exposition). In most of what follows, we confine our attention to equilibria with pure reporting strategies.

Definition 1. *A pure reporting strategy* equilibrium *of this game consists of an audit probability schedule, $p(r)$, reporting strategy, $r(v)$, and posterior updating rule, $f(v \mid r)$, such that*

1. *for every v, $r(v)$ maximizes the manager's expected payoffs,U, for $r \in [0, v]$ and given $p(r)$,*
2. *for every r, $p = p(r) \in [0, 1]$ minimizes the auditor's expected costs, $E(C \mid r)$, where E is the expectation over v given the posterior $f(v \mid r)$, and*
3. *for every equilibrium report, r, $f(v \mid r)$ is the Bayes posterior for the prior, $f(v)$, given the reporting strategy $r(v)$.*

In general, an equilibrium will require that the expected amount of misstatement in a report just be equal to the audit cost[5]. This is because if there is too much expected fraud in a report, the auditor will wish to audit with probability one, in which case, the manager would not issue that report unless he is being truthful and the report would not be misstated. And if there is too little fraud in a report, the auditor will not audit and (if it is a low report) the manager will wish to send that report, thereby increasing the expected misstatement. When reports are misstated just by the audit cost, the auditor will be willing to audit probabilistically; an equilibrium audit policy must then motivate the desired reporting behavior from the manager.

There is an alternate way of modelling this problem. Although an audit occurs after the manager issues his report, in some circumstances it may be possible for the auditor to formulate the audit policy prior to the report. Morton [Mor93] and Sanchez and Sobel [SS93] have analyzed auditing in this case and have found that the following audit policy is optimal:

Definition 2. *A* commitment audit policy *is an audit probability schedule*

$$p(r) = \begin{cases} \frac{1}{M+1} & \textit{if } r < r^* \\ 0 & \textit{if } r \geq r^* \end{cases}$$

[5] This accounts for our decision to use a continuous, rather than discrete, formulation for the value of the firm. In general, with discrete values and reports, the manager must use a random reporting strategy to ensure the expected amount of misstatement is equal to the audit cost, and there will seldom be a pure reporting strategy equilibrium. Because of this, the continuous formulation is in fact more tractable.

for some cutoff report $r^* \in [0, V]$ *which the auditor chooses optimally. When* v *is less than the cutoff, the manager cannot avoid being audited with a probability just sufficient to deter fraud and so will be willing to report truthfully; when* v *is greater than the cutoff, the manager will report the lowest amount,* r^**, which carries no risk of discovery. So the corresponding* commitment reporting strategy *is*

$$r(v) = \begin{cases} v & \text{if } v \leq r^* \\ r^* & \text{if } v \geq r^*. \end{cases}$$

We have called this a commitment policy because it requires the auditor to commit himself to a policy which he will later wish to abandon. To see why, note that the auditor would not be willing to follow this policy after he receives the report, since it calls for an audit of reports which are known not to be misstated, and so is not an equilibrium in our sense. Even if the auditor announced this policy in advance, if the manager knows it can be revised at the time of audit, the manager may not find it credible and would instead predict the auditor will use a policy which satisfies 2. and 3. above. It would be ideal for the auditor to announce a policy, and have it be believed, but then follow a different policy at the time of audit, but this is unlikely to fool a sophisticated and rational reporter who understands the nature of the game.

If there is some mechanism by which the auditor can costlessly commit himself, then the auditor would generally wish to do this, because, according to the results of Morton [Mor93] and Sanchez and Sobel [SS93], he could have committed himself to an equilibrium policy in our sense but chose not to, evidently to do better. However, the plausibility of such mechanisms need to be considered carefully. Theoretically, one could publicly post a large bond with a reliable third party, guaranteeing that the audit policy would be implemented, on penalty of forfeiting the bond. Alternatively, one might argue that long run reputation effects might enable a commitment to audit reports which are truthful (see Schelling [Sch60] for a classic discussion of commitment techniques). However, neither bonding nor public proclamation of the audit policy is typically observed in practice, perhaps because of the difficulty of verifying a probabilistic strategy. Another idea, suggested by Fershtman et al. [FJK91], and by Mookherjee and Png [MP89], is that the audit policy maker could delegate the implementation of the audit to a computer, or to a subordinate with an incentive structure to follow the policy rather than discover fraud. Delegation is very common in practice but, conceptually, it appears to push back the incentive problem one level: what sustains a commitment to the computer program or the incentive structure or how does the policy maker prevent himself from altering the policy at the time of audit? Because of these difficulties with commitment we believe our equilibrium without commitment is plausible in many circumstances.

In addition, a commitment audit policy is not a sensitive forum for exploring the role of information in reporting and auditing. With commitment, whenever a report is audited a misreport is never discovered because the audit is done with sufficient intensity to deter fraud from that report. The auditor does not attempt to extract information from a report and the audit never reveals any new information. Thus the deterrent effect of auditing has overwhelmed any use of information in the audit. In contrast, the sequential equilibrium we use here is designed to explore just these issues. The next section shows that an equilibrium audit policy does require the auditor to use the information contained in the manager's report.

3 Equilibria of Perfect Audits

This section analyzes the equilibria of the perfect audit game. We begin by characterizing an equilibrium in which the manager's reports are very informative. Like the commitment policy, the auditor will be able to infer the value of the firm before auditing, yet unlike commitment, this is not because the report is truthful since the auditor will be unwilling to incur the audit cost to merely verify a truthful report. This audit policy is also qualitatively different from the commitment policy in that lower reports will be audited with strictly higher probability. There are additional equilibria of this audit game, including some which resemble commitment audit policies (although the manager is almost never truthful in his reporting strategy). All of these equilibria have monotone reporting strategies in which the manager's report is nondecreasing in the value of the firm[6]. With a multiplicity of equilibria it is important to select one as most plausible, so this section concludes by showing how to eliminate all but the most informative equilibrium by using a refinement of sequential equilibrium.

In analyzing this game, it is useful to focus on the nature of the manager's reporting strategy, which may be *separating*—the manager makes distinct reports for distinct firm values, or *pooling*—the manager sometimes makes the same report for distinct values. The auditor will use the report to determine the updated value of the firm in deciding whether to audit, so separating reports will give the auditor perfect information about the value of the firm. For a separating report, to induce the auditor to audit, it must be that the amount of fraud in each report is just equal to the audit cost, so this immediately suggests that a separating equilibrium have reporting strategies $r(v) = v - c$. The audit probability schedule must then be chosen to induce this strategy from the manager.

However, for $v < c$, this strategy calls for the manager to make a negative report, which was assumed not to be feasible. Therefore, the perfectly revealing reporting strategy must be modified to allow for pooling at the lowest report, $r = 0$. This report must be audited with positive probability since otherwise the manager would always report 0. With pooling, the auditor will not be able to infer the firm value, but to induce auditing it must still be that

[6] In the appendix, we discuss examples of both nonmonotone and probabilistic reporting strategies.

the average amount of fraud in this report is just equal to the audit cost. So define a lower interval of firm values whose average is equal to c.

Definition 3. *Let $I_0 = [0, v_1]$ be a lower interval such that $E(v \mid v \in I_0) = c$.*

I_0 is the unique lower interval of types which, if all and only types in that interval chose the report $r = 0$, the auditor's expected recovery from auditing $r = 0$ would just equal the audit cost. We will assume that $Ev > c$, since otherwise there will exist only the trivial equilibrium in which it is never worthwhile for the auditor to audit, even when the manager always defrauds the firm of its entire value. Because $f(v)$ is a continuous probability density function, the interval I_0 exists. We can now state

Proposition 1. *There exists an equilibrium (unique as to the audit probabilities of equilibrium reports) in which types $v > v_1$ use the separating strategy $r(v) = v - c$ and types $v < v_1$ report $r = 0$. The equilibrium audit probability schedule is given by*

$$1 - (M + 1)p(r) = \exp \frac{r - (V - c)}{c}$$

for reports $r \in (v_1 - c, V - c]$ and by

$$1 - (M + 1)p(0) = c \exp - \frac{V - v_1}{c}$$

for the report $r = 0$.

Since every report that is sent contains an average of c amount of misstatement, the auditor is indifferent to auditing or not, and is willing to audit with these probabilities. It is straightforward to verify that this audit schedule will induce the required reporting strategy from the manager. The more difficult part of the proof is uniqueness, which relies on an argument by construction using an envelope technique.

This equilibrium is very different from a commitment audit policy since it is a strictly decreasing audit schedule. From an *ex ante* perspective, the lowest reports are most likely to contain fraud in relation to the prior expected value of the firm, and these reports are audited most intensively. Further, in contrast to the commitment audit policy which never discovers a misreport, an audit always discovers some amount of misreport except in the zero probability case that $v = 0$.

There are other equilibria of this game, but the maximally informative equilibrium just described will be the unique one to survive refinement. To show this, we will characterize two additional classes of pure reporting strategy equilibria. The most convenient way to categorize equilibria in signalling games is by the nature of the pooling in the reporting strategy. We begin by asking, when can the maximally separating equilibrium be perturbed by adding some additional pooling? The answer is that, subject to mild conditions, almost any interval partition, P, of the set of possible firm values, $[0, V]$, can be an equilibrium, in which values in each interval in the partition pool by making the same report[7].

Proposition 2. *Suppose there is an equilibrium in which $p(r) < 1/(M+1)$, for all equilibrium reports, r. Then there is an interval partition, P, of $[0, V]$ (in which the intervals may overlap at their endpoints) such that*

1. $I_0 = [0, v_1] \in P$,
2. *for every other I,* $\inf I > E(v \mid v \in I) - c$;
3. *every $v \in I$ reports $r = E(v \mid v \in I) - c$, except for the highest interval for which r may be less than this.*

Conversely, suppose that P satisfies (1) and (2). Then there is an equilibrium in which $p(r) < 1/(M+1)$ for all equilibrium reports and in which the reporting strategy is $r(v) = E(v \mid v \in I) - c$ for $v \in I$.

These conditions can be motivated as follows: First, the partition, P, must consist of connected intervals. This is because the usual "single-crossing" property of the manager's indifference curves holds here, so that if a given manager type prefers a higher report to a lower report, then so do all higher types. Second, the lowest interval I_0 must be an element of the partition for any of these equilibria since, also by the single-crossing property, these are all and only the types who will report $r = 0$. Therefore, there are never any perfectly separating equilibria here since pooling at I_0 is necessary. Third, the auditor must be indifferent in order to choose $0 < p(r) < 1$; so the report for each pool or interval, I, of types is set so that

$$r(I) = E(v \mid v \in I) - c.$$

But all types $v > 0$ will obtain strictly positive payoff since $p(r) < 1/(M+1)$, which requires the final condition that $v > r(I)$, for all $v \in I$.

Audit policies that look similar to the one used under commitment can also be observed in the no-commitment equilibrium, although the reporting strategy must be different from the commitment game in order to induce the auditor to audit. Instead of reporting truthfully in the audit region, in which case the auditor would not be willing to audit, the manager must misstate the report by the amount c. Because the commitment audit policy leaves the manager indifferent among reports in the audit region, $r < r^*$, it is often possible to construct the required reporting strategies which leave the auditor indifferent.

Proposition 3. *Suppose an equilibrium exists in which*

[7] Partition equilibria have been explored in the accounting literature on financial disclosure by Gigler [Gig94], and, most recently Morgan and Stocken [MS98].

(i) $p(r) = \begin{cases} \frac{1}{M+1} & \text{if } r < r^* \\ 0 & \text{if } r \geq r^* \end{cases}$

and the manager's reporting strategy is pure and monotone. Then there is an interval partition P *of* $[0, V]$ *such that*

(ii) $I_0 \in P$
(iii) the highest interval of the partition contains $[V - c, V]$*, and*
(iv) for every interval $I \in P$*,* $\inf I \geq E(v \mid I) - c$*.*

Furthermore, the highest interval is $[r^*, V]$ *and the manager's equilibrium reporting strategy is*

(v) $r(v) = \begin{cases} r^* & \text{if } v \geq r^* \\ E(v \mid I) - c & \text{if } v \in I. \end{cases}$

Conversely, suppose there is an interval partition of $[0, V]$ *which satisfies (ii), (iii) and (iv). Then let* r^* *be the infimum of the highest interval in the partition and there is an equilibrium in which the audit strategy is given by (i) and the reporting strategy by (v).*

This shows that there are many equilibria in which the auditor appears to use a commitment audit policy. As before, when the reporting strategy is monotone, all and only types in the lower interval I_0 will report $r = 0$. At the other end of the range of firm values, the manager will report r^* whenever $v > r^*$. In between, the manager cannot avoid the audit region and is indifferent among all reports. If each interval of types uses the pooling strategy according to (v) (which never requires the manager to overreport if (iv) is satisfied) then the auditor will also be indifferent and be willing to audit according to the commitment policy (i). This is nevertheless quite different from the commitment equilibrium because (iv) and (v) together imply that, except when $v = 0$, the manager always underreports the value of the firm.

In general, the cutoff value that is optimal in the commitment audit policy may not be an equilibrium cutoff value here. However, even if the two audit policies are identical, the auditor will be strictly better off with commitment because, with commitment, the auditor can enforce truthful reporting and ensure no misreporting when $v < r^*$[8]. Without commitment, the manager will almost always misreport when $v < r^*$ and the auditor will recover this fraud only with probability $\frac{1}{1+M} < 1$. Thus, although the audit policies may be identical, the reporting strategies are very different.

This multiplicity of equilibria (the appendix provides examples which show that there are also equilibria with nonmonotone and with mixed reporting strategies) can be resolved by showing that the maximally separating equi-

[8] Although the manager is indifferent between truthful and misreporting an infinitesimally higher audit probability will make the manager strictly prefer to report truthfully.

librium of Proposition 1 is the most plausible[9]. All equilibria of this game involve some pooling in the manager's report (unlike the model of Reinganum and Wilde [RW86]) and also contain out-of-equilibrium reports which are never sent by the manager no matter what value he may observe. These off-equilibrium reports are often of crucial importance since the auditor's response to these reports are precisely what prevents them from being sent and make the equilibrium reports rational for the manager. Sequential equilibrium places almost no restrictions on how the auditor can interpret out-of-equilibrium moves, and so he can interpret them in fairly silly ways in order to make an otherwise incredible response to prevent the move from being made. Refinements of sequential equilibrium generally focus on how to interpret out-of-equilibrium moves—what sense will the auditor make if he observes a report which, according to the equilibrium, should never have been sent—and place additional restrictions on these out-of- equilibrium beliefs by asking for the most reasonable interpretation to place on reports that should not have occurred.

To illustrate the qualitative differences between the equilibria we now present some numerical examples. Suppose $\tilde{v}$ is uniformly distributed on the interval $[0, 100]$, suppose that the verification cost $c = 20$ and that the penalty parameter $M = 10$. We will now construct partition equilibria of the type described in Propositions 2 and 3.

Example 1. For these parameters, $v_1 = 40$. An example of a hypothetical three element partition is given by the following: $I_0 = [0, 40]$, $I_1 = [40, 70]$ and $I_2 = [70, 100]$. All types $v \in I_0$ report $40 - c = 0$. All types $v \in I_1$ report

[9] We have received queries about mixed reporting strategies like the ones found in Crawford and Sobel [CS82]. We should emphasise that our model is not a cheap talk model like Crawford-Sobel. In Crawford-Sobel, messages are 'cheap talk' *i.e.,* messages are costless. In our model, messages *are* costly: a report of v involves making a payment of v. In addition, if the true type of the manager is v, he prefers not to report more than v, while Crawford-Sobel make no such restriction, and over-reporting might occur in equilibrium in their model. The following is obviously not an equilibrium (as has been suggested): Take any interval $I_i = [a_i, b_i]$ in the partition equilibrium discussed in the paper. Let the manager randomise over $a_i - c$ to $b_i - c$. Any reports in this range are supposed by the auditor to come from I_i. This is impossible to sustain as an equilibrium with non-degenerate mixed auditing strategies, since $E[v \mid I_i] - r = c$, for such a mixed strategy to be in equilibrium, and this cannot be true for two distinct values of the report r. (Everything else in the expression above remains the same..) It is also unclear how the manager with different values of v can be indifferent among such reports, because it might involve reporting more than the actual value–a negative payoff.

With a continuum of types, the restriction to pure reporting strategies is a natural one to make, though the pathological examples with non-monotone reports shows there could be mixed-strategy equilibria. The "disturbed" game interpretation of mixed strategies (due to Harsanyi) in fact uses pure strategies with a continuum of types to purify mixed strategies.

$55-c=35$. All types $v \in I_2$, the equilibrium report is $85-c=65$. All reports are audited with probability $p(r) \leq \frac{1}{11}$.

Example 2. In fact this is *not* the only three element partition. Consider $I_0 = [0, 40]$, $I_1 = [40, 76]$ and $I_2 = [76, 100]$. All types $v \in I_0$, report $40 - c = 0$. For $v \in I_1$, report $58 - c = 38$. For $v \in I_2$, the equilibrium report would be $88 - c = 68$.

Example 3. One can similarly build a four element partition $I_0 = [0, 40]$, $I_1 = [40, 60]$, $I_2 = [60, 80]$, $I_3 = [80, 100]$ for the same set of parameters.

Example 4. For all these equilibria, the first and last elements of the partition are fixed. If we just restricted ourselves to partitions where the other elements were of equal length, one equilibrium induces the partition $I_0 = [0, 40]$, $I_1 = [40, 60]$, $I_2 = [60, 80]$, $I_3 = [80, 100]$ and the corresponding equilibrium reports are $\{\{0\}, \{30\}, \{50\}, \{70\}\}$. Corresponding audit probabilities are any p such that $p < \frac{1}{11}$ for all reports.

There may be different equilibria for the same parameter set. We will now present the 'maximally separating equilibrium' described in Proposition 1.

Example 5. This equilibrium induces the partition $I_0 = [0, 40]$, $I_1 = [40, 100]$. Corresponding equilibrium reporting strategies are $\{0\}, \{v - 20\}$. The corresponding audit probabilities are respectively 0.000387148 if 0 is reported, and $\frac{1-exp(0.05r-4)}{11}$ for reports r in the upper tail. We obtain $p(0) = 0.000387148$ by solving $1 - 11p(0) = 20exp(-\frac{100-40}{20})$.

Example 6. In Proposition 3, we described the 'commitment like' partition equilibria: an example follows. This equilibrium induces the partition $I_0 = [0, 40]$, $I_1 = [40, 60]$, $I_2 = [60, 80]$, and $I_3 = [80, 100]$. Corresponding equilibrium reports are $\{\{0\}, \{30\}, \{50\}, \{80\}\}$, with corresponding audit probabilities $\{\{\frac{1}{11}\}, \{\frac{1}{11}\}, \{\frac{1}{11}\}, \{0\}\}$ respectively.

Example 7. We now show that for the same parameter values, one can construct two distinct equilibria of the type described in Proposition 2, one of which has a 5 element partition and the other has a 4 element partition, with the additional feature that the 'finer' partition (the one with 5 elements) refines the 'coarser' 4 element partition. If we assume the same parameter values as before, one can consider an equilibrium which induces the 4 element partition $I_0 = [0, 40]$, $I_1 = [40, 60]$,$I_2 = [60, 80]$,$I_3 = [80, 100]$, with corresponding equilibrium reports $\{\{0\}, \{30\}, \{50\}, \{80\}\}$ and another equilibrium which induces the 5 element partition $I_0^{'} = [0, 40]$, $I_1^{'} = [40, 60]$, $I_2^{'} = [60, 70]$, $I_3^{'} = [70, 80]$, $I_4^{'} = [80, 100]$, with corresponding equilibrium reports $\{\{0\}, \{30\}, \{45\}, \{55\}, \{80\}\}$. As our previous examples illustrate, partition equilibria can not, in general, not be ranked in terms of 'fineness'.

The above examples illustrate the magnitude of the issue of multiple equilibria that we face, and lead us to the next issue, that of refinements.

Although there are many refinements available in the literature, we shall use "Divinity" by Banks and Sobel [BS87] and reformulated as "D1" by Cho and Sobel [CS90], who show that it is generically equivalent in this sort of signalling game to the "strategic stability" of Kohlberg and Mertens [KM86], the most powerful refinement criterion available. It is difficult to characterize strategic stability in infinite games like ours, so this equivalence may not hold here. Nevertheless, it gives special plausibility to the use of D1. Also, for a general class of signalling games, Cho and Sobel [CS90] have shown that D1 selects a unique equilibrium, a result we confirm below.

In our context, D1 can be described as follows: the auditor observes an out-of-equilibrium report, r, and is considering whether manager type v may have sent the report. The answer depends on the relative strength of the manager's incentives to send that report and what he speculates may happen if he sends this report rather than his equilibrium report. If there is some other type v' who would be willing to issue r under a wider range of possible responses by the auditor than would v, then D1 requires that the auditor never believe that v could send report r. More formally,

Definition 4. *For any equilibrium, let $A(v)$ be the set of audit probabilities for r for which type v would either weakly or strictly prefer the report r to his equilibrium report. Let $B(v')$ be the set of audit probabilities for r for which some type v' would strictly prefer the report r to his equilibrium report. If $A(v) \subset B(v')$, then type v' has stronger incentives to deviate from his equilibrium report than v does, because v' would strictly prefer a larger set of possible auditor responses than v weakly prefers. In such a case, D1 requires that the auditor cannot place any weight on the conjecture that type v sent the off equilibrium report. In particular, the equilibrium satisfies D1 if the Bayes posterior for this report r and type v is zero: $f(v \mid r) = 0$.*

Because the manager always prefers a lower audit probability (and because it is continuous and monotonic in his expected payoff), this condition can be simplified somewhat. All types who are deterred by lower probabilities of audit are also deterred by higher probabilities. For every type, calculate the audit probability for an out-of-equilibrium report which would leave the manager indifferent between that report and his equilibrium report. Then, if that report is observed, the auditor must believe it was sent by the type(s) who have the largest such probability. An equilibrium survives D1 if all of the off-equilibrium beliefs satisfy this criterion, and most importantly, the audit probabilities specified for these out-of-equilibrium reports are optimal given these beliefs.

We can now show that only the maximally informative equilibrium of Proposition 1 survives D1. All of the others fail because D1 prevents the auditor from adopting beliefs which allow him to audit with sufficient probability to deter some manager types from making out-of-equilibrium reports.

Proposition 4. *The only equilibrium to survive D1 is the maximally informative equilibrium of Proposition 1.*

The idea of the proof is to consider the reports made by any two adjacent intervals in the pooling partition. The reports in between are out-of-equilibrium. For an out-of-equilibrium report, r', sufficiently close to the higher report, r, D1 requires that the auditor believe it was sent by the type $\inf I$ on the lower boundary of the upper adjacent interval, I, since this type has the strongest incentive to send such a report. But if $r - E(v \mid I) = c$ so that the auditor is willing to audit r, then $r' - \inf I < c$, if r' is sufficiently close to r, and the auditor is not willing to audit r'. But this cannot be an equilibrium because the manager would then prefer the lower r' which is not audited to the higher report r which he is supposed to make.

This argument rules out the equilibria of Proposition 2, where there is much pooling among high types, and also the commitment-like equilibria of Proposition 3, where reports immediately below the cutoff report are out-of-equilibrium. It does not rule out the maximally informative equilibrium. Although all reports $r' \in (0, v_1 - c)$ are never observed in equilibrium, D1 requires that the auditor believe that v_1 sent such reports. But in this separating equilibrium, $v_1 - r(v_1) = c$, so $v_1 - r' > c$ for lower reports and the auditor will wish to audit with probability one, which is what is required to prevent the manager from sending those reports.

4 Comparative Statics

Since we have selected the maximally separating equilibrium as the most reasonable solution to this game, we will concentrate on this equilibrium to discuss the empirical implications of the analysis.

An *increase in the penalty rate*, M, will uniformly decrease all the audit probabilities, since with higher penalties, one need audit less often to obtain the same reporting behavior. However, M has no effect on the reporting strategy. This is because the manager's reporting strategy must be chosen to leave the auditor indifferent between auditing or not, and the auditor is not directly affected by M. This implies that the penalty rate will not affect the initial amount of misreporting, but will affect the average amount discovered after an audit—a higher penalty rate will increase the expected amount of misreporting remaining after an audit.

Recall that the auditor's costs are $C = v - r + p(r - v + c)$. Given the manager's reporting strategy, the last term is expected to be zero, so the auditor's costs are just the initial amount of underreporting, which is unaffected by M. Although we have held the auditor's fee revenue from auditing constant in this game, this suggests that audit fees will also be unaffected by M if they are determined by the total costs of the auditor. The manager's expected payoff, $U = [1 - p(M + 1)](v - r)$, is also unaffected by M. Although the audit probability will change with M, there is no net effect on the term $[1 - p(M + 1)]$. There is also no effect on the reporting strategy. In sum, changes in the penalty rate will affect the audit strategy, but very little else.

A *change in the audit cost*, c, will have more consequential effects. As c increases, there will be uniformly less auditing and more misreporting, so the expected costs of the auditor and the expected payoffs to the manager will both increase. Therefore, audit fees will increase with c. The reason c plays, a more fundamental role in the model than does M, is that M enters the model only through the term $1 - (M + 1)p$, in the manager's expected payoff function, so p can adjust to accommodate any changes in M without affecting any other aspect of the solution. On the other hand, c determines the amount of misreporting, as well as the cutoff value, v_1, for types that will report $r = 0$.

There are two ways in which a change in the *prior distribution of firm values* may affect the equilibrium: through a change in v_1 or a change in V. Holding V constant for the moment, if it becomes more likely that the firm's value is larger (in the precise sense of a decrease in the conditional expectation of the lower interval $E(v \mid v \in I_0)$, a condition which is not in general equivalent to first order stochastic dominance) then v_1 must increase. The only effect on the equilibrium is that a larger interval of firm values will report $r = 0$, and $p(0)$ will decrease to attract this larger interval. For values who continue to make strictly positive reports, there is no change in either the reporting strategy or audit probabilities. Since the expected amount of misreporting at $r = 0$ must still be equal to c, there is no change in the total amount of misreporting, or in the expected costs of the auditor. Less fraud will be discovered, however, because $p(0)$ has decreased.

If the upper bound, V, of the support increases, but without altering the conditional expectation around the lower interval, $[0, v_1]$, then the audit probability schedule will increase, so that $p(r)$ will be higher for every r. This will decrease the manager's expected payoff, but the expected amount of misreporting and the auditor's expected costs will be unchanged at c. It often appears that the highest reports are audited most intensively in practice, particularly in a tax audit context. This may be one explanation of this practice because, even though the wealthy taxpayer reports higher income than an indigent, the auditor may have very different priors for the two taxpayers(This is also the explanation in Reinganum and Wilde [RW86]).

5 Imperfect Audits and Audit Effort

In this section, we examine how this model may be generalized to include an effort choice for the auditor. To this point, the audit was assumed to be perfect and would perfectly reveal the true value of the firm. The only effort choice concerned the probability with which the auditor would undertake an audit. Since audits are seldom perfect in fact, auditing will frequently be sequential, in which further investigation is done after an initial examination of the manager's report. The details of this process are complicated and will surely vary according to the circumstances, so we wish to make our first step in the analysis of sequential audits as simple as possible. One natural assumption is that an audit is imperfect and will discover a misreport only with some probability less than one, which might be described as the reliability of the audit technology. If the auditor audits only once and fails to discover a misreport, he cannot be sure that the report was correctly stated; but it is now reasonable to repeat the audit since the probability of discovering a misreport (if one exists) is strictly increasing in the number of repetitions. Of course, the total audit cost should also be increasing, so this will enable us to examine the trade offs the auditor faces in choice of audit effort, that is, in the number of repetitions of the audit.

Suppose π is the probability that an audit will discover a misreport if one exists. Therefore, with probability π, a single audit will publicly reveal the value of the firm and the auditor can cease any further investigation; but with probability $1-\pi$, the audit will not discover any information to disprove or confirm the manager's report. The auditor's decision problem now reflects the imperfect reliability of the audit technology. Consistent with our earlier approach that the auditor cannot commit in advance to an audit policy, we assume that at each repetition the auditor will decide whether to investigate further[10]. This results in an infinite dynamic programming problem in which, at each stage, i, the auditor will choose the probability of auditing, p_i, to minimize his expected current and future costs. Let E_i be an expectation with respect to v based on whatever information the auditor has acquired up to stage i. If he audits report r at stage i he will incur the audit cost of c. With probability π he will learn the value of the firm and will stop, but with probability $1 - \pi$, he will not discover the value of the firm, and he will face

[10] An alternative to this sequential decision problem might be called batch auditing in which the auditor decides in advance of any auditing, but after observing the manager's report, how many repetitions to make, somewhat in the way sample sizes are often calculated. This batch approach to repeated auditing may not constitute a credible audit policy since the auditor may wish to change the batch size after it is partially collected. Reinganum and Wilde's [RW86] assumption of an audit cost which is convex in the probability of discovery appears to be equivalent to batch auditing. Other possible interpretations of their formulation are also possible, including the size of the audit team assigned to a particular task.

the costs of proceeding. Of course, he will also face these future costs if he does not audit[11].

Letting C_{i+1} be the expected future costs beyond stage i, the auditor will choose p_i to minimize

$$C_i = E_i p_i[c + \pi \cdot 0 + (1-\pi)C_{i+1}] + (1-p_i)C_{i+1}$$

If the auditor ceases to audit at stage N and beyond, his cost becomes

$$C_N = E_N(v-r).$$

Since the result of each audit is independent, the probability of *not* discovering the value of the firm after auditing the report r with probabilities $p_i(r)$ is $\prod_{i=0}^{\infty}(1-p_i(r)\pi)$, so the probability that the value *will* be discovered is

$$p(r) = 1 - \prod_{i=0}^{\infty}(1-p_i(r)\pi).$$

The payoffs to the manager are unchanged from the perfect audit game, except that the manager cares now only about the probability of discovery, and not the probability or intensity of audit per se:

$$U = (1-p)(v-r) - pM(v-r) = [1-p(M+1)](v-r).$$

The manager will choose his report to maximize U given $p(r)$.

We note two features of this equilibrium which simplify the auditor's decision problem. First, an audit either reveals the firm's value or not. If it does, then auditing and the game terminates. If it does not, then the auditor gets *no additional information about* v[12]. Therefore the auditor's information at every stage that the game continues is identical to his information immediately after the manager's report: $E_i(\cdot) = E(\cdot \mid r)$. Second, suppose in any stage that the auditor strictly desires to audit. If a misreport is not discovered in that stage, then, since the auditor has gained no additional information about v, he will also strictly desire to audit in the next stage. This will continue in every stage until the probability of discovery approaches one, which cannot be an equilibrium since the manager would not misreport when he is certain to be discovered. Therefore, in equilibrium, whenever the auditor is willing to audit a report even once, it must be that he is indifferent to auditing in

[11] This implicitly assumes a strict liability rather than negligence standard for the auditor, since he will be subject to penalty whenever he fails to discover a misreport, no matter how intensively he audited. It would be interesting to explore a negligence standard in which the auditor is penalized only if he fails to collect sufficient competent evidence. We are not yet sure how to model such standards of evidence. Also, in practice, auditors are liable to be sued whenever they do not find a misreport, so the strict liability regime may be the more plausible.

[12] This is an important aspect of the specification.

every stage, and being indifferent, he can ignore the future costs. He will then choose p_i to minimize

$$\begin{aligned} C_i &= Ep_i[c + (1-\pi)(v-r)] + (1-p_i)(v-r) \\ &= E(v-r) + p_i[c - \pi E(v-r)]. \end{aligned}$$

This shows that the auditor's dynamic programming problem is, in this game, equivalent to a myopic, one period problem. With this we can now prove the following:

Proposition 5. *Suppose $p(r), r(v)$ are equilibrium auditing and reporting strategies when the audit is perfect and the audit cost is c'. Let $p_i(r)$ be any audit probabilities such that for every r*

$$p(r) = 1 - \prod_{i=0}^{\infty}(1 - p_i(r)\pi).$$

Then $p_i(r), r(v)$ constitute a sequential sampling equilibrium when the audit cost is $c = c' \cdot \pi$.

Since this proposition shows that every imperfect audit can be translated into a perfect audit, the same selection principles can be used to choose the maximally informative equilibrium as the uniquely plausible outcome of the model. It also shows that, in a qualitative sense, the basic model with perfect auditing is very robust to the incorporation of audit effort. Aside from observing repeated audits, imperfect auditing is essentially a change in the audit cost. It is also apparent that many different combinations of $p_i(r)$ can result in the same $p(r)$. It is natural to focus on the monotonic audit strategy in which the auditor audits $n(r)$ times with probability one until the last which is audited with probability $p_n(r)$. These are uniquely determined by the probability of discovery

$$p(r) = 1 - (1-\pi)^{n(r)}(1 - p_n(r)\pi).$$

With this convention, as the system becomes more reliable, the number of repetitions declines.

6 Conclusion

In this paper, we have examined a simple model in which the strategic interplay between commitment and informational asymmetry can be studied. This is an alternative to the more common contracting approach in accounting which assumes commitment and public verifiability. The inability of the auditor to commit leads to a wide variety of equilibrium auditing and reporting behavior. It is not enough for the auditor to merely specify an audit policy,;this policy must also be consistent with expectations about the manager's reporting strategy and, in particular, about the interpretations formed when unexpected, out-of-equilibrium reports are observed. These additional restrictions imposed on the auditor have surprising implications in permitting more rather than fewer equilibria. Further, in the equilibria where the auditor uses an audit policy similar to the one used in the commitment equilibrium but the manager never uses a truthful reporting strategy, these restrictions on the auditor can change the manager's behavior but not the auditor's.

By examining the plausibility of various out-of-equilibrium expectations, we were led to a unique equilibrium in which the manager always misreports the value of the firm and this misreport is sometimes discovered. This is the polar opposite of the commitment model where the value of the firm is always truthfully reported when it is below the cutoff and the audit never discovers a misreport. Casual empiricism suggests that both results are extreme, since audits sometimes, but not always, discover a misreport. These extreme results are generic in simple models, and to obtain the middle ground it may be necessary to depart from strict rationality assumptions.

Another difference between the commitment and no commitment models is that comparatively little use is made in the latter of the probability distribution of types. The upper bound and lower tail of the distribution have marginal effects on the equilibrium, but otherwise, the reporting and audit strategies are largely independent of the distribution. The distribution is determined by institutional features and the equilibrium's robustness here is an attractive feature.

In both models, the informational role of the audit is relatively minor. To a large extent, it is the report that conveys information on the value of the firm. In the commitment model this is always true and the audit plays an exclusively deterrent role. Many of the equilibria without commitment have fairly uninformative reports, which leaves room for the audit to discover something, but we have shown that the equilibrium in which the reporting strategy is maximally informative is the most plausible in this case as well. It appears that the informational aspect of auditing is subsidiary to its deterrent role.

Comparative statics on the maximally informative equilibrium highlight the importance of the audit cost. A change in that cost has pervasive effects on both the audit policy and reporting strategy, whereas changes in the penalty rate on the manager or the prior distribution of firm values are of less consequence.

We have also examined a generalization to imperfect auditing and audit effort. Formally, the auditor's problem becomes an infinite dynamic program since he may now wish to repeat the audit to obtain greater confidence that the audit was reliable. With considerable relief, we were able to simplify the problem to a static program and show that an imperfect audit was essentially equivalent to an increase in the audit cost of a perfect audit. In addition to showing how audit effort responds in equilibrium to the reliability of the audit, this robustness gives greater credibility to the basic game.

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Appendix

Because there are many equilibria of this game, it is useful to begin by stating some conditions that will be true of all equilibria.

Lemma 1. *For any equilibrium,*

1. $p(r) > 0$ *when* $r = 0$.
2. $p(r) = 0$ *for* $V - c < r \leq V$.
3. $p(r) \leq 1/(1+M)$ *for any report chosen by the manager.*
4. $p(r)$ *is nonincreasing among the reports that may be chosen by the manager.*

Proof. (1) If $p(0) = 0$, then the manager will always choose $r = 0$, since this maximizes fraud and also has no risk of discovery. But then the posterior given $r = 0$ will be identical to the prior and the expected recovery from auditing will be $Ev - c > 0$. So, the auditor will wish to audit with probability one, a contradiction.

(2) Since the manager can never report more than the value of the firm, the maximum amount that can be recovered from a report $r \in (V - c, V]$ is $V - r < c$. Therefore, the expected recovery from such reports is less than the audit cost and the auditor will never audit.

(3) A probability of audit of $1/(1 + M)$ is just sufficient to deter all underreporting, so if the manager chooses a report with $p(r) > 1/(1+M)$, he must be reporting truthfully. Consequently, there will be nothing to recover in such a report and the auditor will prefer to choose $p(r) = 0$.

(4) A lower report implies that the manager is appropriating more of the value of the firm to his own use. If he can obtain more rents from the firm at a lower risk of discovery, he will never issue the higher report, contradicting the assumption that the manager will sometimes issue the higher report.

Proof of Proposition 1:

Since every type $v > v_1$ misreports by exactly c, and since all types $v < v_1$ report $r = 0$, which is an average misreport of c, the auditor is willing to audit with these probabilities. In turn, it is straightforward to verify that it is optimal for the manager to make these reports when faced with this audit policy.

To prove that this audit policy is necessary given this maximally separating reporting strategy, we construct the unique audit probability schedule that will induce this reporting behavior using a general technique based on the envelope theorem. Define the maximum value function of the manager's reporting problem when faced with some audit probability schedule, $p(r)$:

$$u(v) = \max_r [1 - p(r)(M + 1)](v - r) \mid 0 \leq r \leq V.$$

This is the manager's indirect utility as a function of the firm value. By the Maximum Theorem of Berge [Ber63], u is continuous in v and its total derivative is equal to the partial of U with respect to v, whenever it exists:

$$u'(v) = \frac{\partial}{\partial v}[1 - p(r(v))(M+1)](v - r) = 1 - p(r(v))(M+1) = \frac{u(v)}{v - r(v)}.$$

Using the boundary condition that the highest type is never audited, $p(r(V)) = 0$ or $u(V) = V - r(V)$. The solution to this differential equation is

$$u(v) = [V - r(V)] \exp - \int_v^V \frac{1}{t - r(t)}\, dt$$

for $v > 0$, and for continuity let $u(0) = 0$. When $r(v) = v - c$, then

$$u(v) = c \exp \frac{v - V}{c} = [1 - (M+1)p(r(v))]c.$$

Substituting $v = r + c$ yields

$$1 - (M+1)p(r) = \exp \frac{r - (V - c)}{c}.$$

To find $p(0)$, we use the fact that v_1 is indifferent between, $r = 0$ and $r = v_1 - c$

$$[1 - (M+1)p(0)] = u(v_1) = c \exp - \frac{V - v_1}{c}$$

and this concludes the proof.

Proof of Proposition 2:

For the first half of the Proposition, suppose $p(r)$ is the audit policy in this equilibrium. Since $p(r) < 1/(M+1)$ for all equilibrium reports r, every type $r > 0$ must obtain strictly positive expected utility. This implies that $p(r)$ must be a strictly decreasing function of equilibrium reports, since otherwise the manager could obtain higher expected payoff by making a lower report without incurring any increased probability of audit.

Let $I(r)$ be the set of types who are willing to choose r:

$$I(r) = \{v \mid \text{for all } r', [1 - (M+1)p(r)](v - t) \geq [1 - (M+1)p(r')](v - r')\}.$$

We will show that $I(r)$ must be a connected interval. Let v and $v'' \in I(t)$ and consider $v'' > v' > v$. If r' is not an equilibrium report, then it cannot be preferred by v' to r, so suppose that r' is an equilibrium report and further that $r' < r$. Since v prefers r to r', some algebra shows that $v' > v$ does also:

$$\begin{aligned}&[1-(M+1)p(r)](v'-r)-[1-(M+1)p(r')](v'-r')\\&\quad=[1-(M+1)p(r)](v-r)-[1-(M+1)p(r')](v-r')\\&\qquad+(M+1)(v'-v)[p(r')-p(r)]\\&\quad\geq\ 0\end{aligned}$$

since v prefers r, $v' > v$, and $p(r)$ is decreasing for equilibrium r. An analogous argument shows that r' will not prefer any $r' > r$ since v'' prefers r and $v > v'$. Thus $I(r)$ is a connected interval. These inequalities also show that if v is indifferent between distinct equilibrium reports r and r', then $v' > v$ strictly prefers one or the other. Thus, two distinct intervals can overlap at most at a singleton.

Since, in an equilibrium, every v must have a maximizing report, every v is in some $I(r)$. Therefore, the set of all $I(r)$, for equilibrium r, form a partition of $[0, V]$, except that the endpoints of an interval of positive length may overlap with its neighbor. By the Maximum Theorem of Berge [Ber63] the maximum value function

$$u(v) = \max_r\ [1-(M+1)p(r)](v-r)\ |\ \ 0 \leq r \leq r$$

is continuous in r, so that such an endpoint does in fact overlap with the neighboring interval and that type is indifferent between the reports of the two intervals. Types in the interior of an interval are in no other interval and so strictly prefer a unique report.

Since $p(r)$ is strictly decreasing among equilibrium reports, only the highest interval can have an audit probability of zero. All other intervals must have a strictly interior audit probability, which requires that the auditor be indifferent:

$$c = E(v \mid v \in I(r)) - r,$$

for all equilibrium reports r. This may hold as a weak inequality for the highest interval. This proves (iii), as well as (i) when $r = 0$.

To prove (ii), recall that whenever $v > 0$, the manager must receive a strictly positive expected payoff and therefore must report less than the full value of the firm. Thus, for the lower endpoint of each interval

$$\inf I(r) > r \geq E(v \mid v \in I(r)) - c,$$

and (ii) is proved. This shows necessity and completes the first half of the proof.

We now show that these same conditions are sufficient for a partition to be the reporting pools of an equilibrium. The proof is by construction and uses an envelope argument, which is shown to be equally applicable to pooling as well as separating equilibria.

For a interval partition P of $[0, V]$, let $I(v) \in P$ be the interval which contains v. Let $r(v)$ be the pure reporting strategy

$$r(v) = E(v' \mid v' \in I(v)) - c.$$

Since $I(v)$ is an interval, $r(v)$ is a nondecreasing step function; by (ii), $\inf I(v) > r(v) > 0$ for $v > 0$, and by (i), $\inf I(0) = 0 = t(0)$, so reports are nonnegative and no type is required to report an amount greater than the firm value. Note also that $r(V) \leq V - c$.

We can now construct a maximum value function, $u(v)$, for the manager using the envelope condition that the total derivative is equal to the partial derivative of the manager's optimal objective function with respect to v. If $r(v)$ is to solve the reporting problem, then the value function must be

$$u(v) \equiv [1 - (M+1)p(r(v))][v - r(v)]$$

and the envelope condition is that

$$u'(v) = \frac{\partial}{\partial v}[1 - (M+1)p(r(v))][v - r(v)].$$

We specify as boundary condition that the highest type is never audited, $p(r(V)) = 0$ or $u(V) = V - r(V)$. The solution to this differential equation is

$$u(v) = [V - r(V)] \exp - \int_v^V \frac{1}{t - r(t)}\, dt$$

for $v > 0$, and for continuity let $u(0) = 0$. The remainder of the proof consists of constructing an equilibrium which yields this $u(v)$ when the manager reports $r(v)$.

To specify audit probabilities for equilibrium reports, construct $p(r(v))$ so that if v chooses $r(v)$ he attains $u(v)$:

$$[1 - (M+1)p(r(v))][v - r(v)] = u(v).$$

These audit probabilities are strictly less than 1/(M+1), since $u(v)/[v - r(v)] > 0$. For reports off the equilibrium path, let

$$p(r) = \begin{cases} 1 & \text{if } r < V - c \\ r0 & \text{otherwise.} \end{cases}$$

The auditor is willing to choose these probabilities since, for equilibrium reports, $r(v)$ was constructed to leave the auditor indifferent. Probability assessments for out-of-equilibrium reports can easily be constructed so that when $r < V - c$, the auditor believes it is some type $v \geq r + c$ and wishes to audit; when $r > V - c$, the auditor believes it is some type $v \leq r + c$ and prefers not to audit.

Given these audit probabilities, we must show that $r(v)$ is a maximizing report for the manager. He will not choose any off-equilibrium report that is audited with probability zero since there is the lower equilibrium report $r(V)$ that is also never audited. He will not choose any other off-equilibrium report since these are audited with probability one. Therefore, it remains only to show that manager v prefers $r(v)$ to any other equilibrium report $r(v')$:

$$u(v) \geq [1-(M+1)p(r(v'))][v-r(v')].$$

Using the definition of $u(v')$ this is equivalent to

$$\frac{u(v)}{u(v')} \geq \frac{v-r(v')}{v'-r(v')}$$

or,

$$\exp - \int_{v'}^{v} \frac{1}{t-r(t)}\,dt \geq \exp - \int_{v'}^{v} \frac{1}{t-r(v')}\,dt.$$

But this last inequality holds because $r(v)$ is a nondecreasing step function. This proves sufficiency and completes the proof.

Proof of Proposition 3:

Since the audit probability $1/(M+1)$ gives the manager an expected payoff of zero, if (i) is the audit strategy, then for $v > r^*$, the manager will report r^*, the lowest report that is not audited, and for $v \leq r^*$, will be indifferent among all reports $r \in [0, v]$. Therefore, the manager will report r^* if v is in the highest interval $I^* = [r^*, V]$. From Proposition 1(ii), reports $r \in (V-c, V]$ will never be audited, so it must be that $r^* \leq V - c$ and this proves (ii). Also, according to (i), the auditor cannot strictly prefer to audit r^*, so $c \geq E(v \mid [r^*, V] - r^*$, and (iii) holds for $I = I^*$.

If the reporting strategy is monotone, the set of firm values for which the manager issues the same report must be a connected interval. Since $v = 0$ is constrained to issue report $r = 0$, the set of values which issue that report must be the interval $I_0 = [0, v_1]$, in which $E(v \mid I_0) = c$, in order to induce the auditor to audit $r = 0$. This proves (i) and (ii) for $I = I_0$.

For other intervals, I, the reports issued when the firm value is in I will be audited probabilistically, so the expected misstatement must equal the audit cost, $E(v \mid I) - r = c$, when r is reported by the values in I. This establishes the reporting strategy for $I \neq I^*$. Finally, the manager is constrained never to report more than the value of the firm, so if r is reported by the values in I, this reporting strategy implies that $r = E(v \mid I) - c \leq \inf I$ and (iii) is proved.

It is straightforward to verify the second half of the Proposition so its proof is omitted.

Example of Nonmonotone and Mixed Reporting Strategies:

Proposition 3 shows that there are an extremely large number of well behaved step function equilibria: subject to mild conditions, any pooling of types by reporting strategy into connected intervals can be an equilibrium. However, even these conditions are not sufficient, since by relaxing the requirement that reporting strategies be monotone, we can also generate an equilibrium in which the pools are not connected intervals and can then generate mixed reporting strategy equilibria.

Example 8. Let $V = 5$ and $c = 1$. Consider the nonmonotone reporting strategies for the nonconnected pools:

$$r(v) = \begin{cases} 0 \text{ if } v \in [0,1] \cup [2,3] \\ 1 \text{ if } v \in (1,2) \cup (3,4) \\ 4 \text{ if } v \in [4,5] \end{cases}$$

and audit strategy

$$p(r) = \begin{cases} 1/(M+1) & \text{if } r < 4 \\ 0 & \text{otherwise.} \end{cases}$$

Suppose v is uniformly distributed within each of these five intervals $[n-1, n]$ but that the probability mass of each interval, q_n, may differ. Set the expected recovery from auditing $r = 0$ and $r = 1$, respectively, to be equal to the audit cost

$$\frac{(.5q_1 + 2.5q_3)}{(q_1 + q_5)} = 1$$

$$\frac{(1.5-1)q_2 + (3.5-1)q_4)}{q_2 + q_4)} = 1,$$

or

$$q_1 = 3q_3$$

$$q_2 = 3q_4$$

respectively. Clearly, these can be chosen small enough so that the auditor is willing to audit $r = 0$ and 1, and there is some positive residual probability for q_5. The out-of-equilibrium beliefs necessary to support this as an equilibrium can be specified easily enough so that any report not used in equilibrium is audited with probability 1.

However, in a preview of the argument used in Proposition 4, these equilibria do not survive plausible restrictions on beliefs off the equilibrium path. Suppose a report of 3.99 is made and audited with a probability p.Obviously, this can only be from types in [3.99,5] since all other types will get a negative payoff even if $p = 0$. Consider some $v \in (4, 5]$. The equilibrium payoff for these types is $v - 4$, since a report of 4 is not audited. Therefore such a type will be indifferent between deviating and playing equilibrium if $(v - 3.99)(1 - (M+1)p) = v - 4$. The quantity $\frac{v-4}{v-3.99}$ is increasing in v, therefore $v = 4$ will strictly prefer deviating for values of p for which higher types would be indifferent, so that the deviation to 3.99 can only come from [3.99,4]. Given this, 3.99 will not be audited, so this equilibrium does not survive D1 (see the proof of Proposition 4) below.

Example 9. Let everything be as in Example 1 and assume specifically that $q_1 = q_2 = .10, q_3 = q_4 = .30$, and $q_5 = .20$. The proof of Proposition 4 shows that there is also an equilibrium with monotone reporting strategies and connected pooling intervals. In particular, it is

$$r(v) = \begin{cases} 0 \text{ if } v \in [0,2] \\ 2 \text{ if } v \in [2,4] \\ 4 \text{ if } v \in [4,5] \end{cases}$$

with the same audit strategy as in Example 8. This is an equilibrium because v is uniform on each of the above three intervals, so $E(v \mid v \in [0,2]) = 1$ and $E(v \mid v \in [2,4]) = 3$.

To construct a mixed reporting strategy equilibrium, suppose the manager randomizes among the reporting strategies of these two equilibria according to the flip of a coin, i.e., independently of v. If the auditor observes a report of 1 or 2, he knows which equilibrium is being played, but not if he observes 0 or 4. In either equilibrium, he is willing to audit reports less than 4, so he is still willing even without being able to infer the equilibrium.

Proof of Proposition 4:

We first show that D1 eliminates the pooling equilibria of Proposition 2, in which the reporting strategy is monotonic and the equilibrium audit probabilities are strictly decreasing. Let $r_n < r_{n+1}$ be two adjacent equilibrium reports, with audit probabilities $p_n > p_{n+1}$, sent by types in the intervals $[v_n, v_{n+1}]$ and $[v_{n+1}, v_{n+2}]$, respectively, with $v_n < v_{n+1} < v_{n+2}$. Consider an out-of- equilibrium report $r \in (r_n, r_{n+1})$.

The largest audit probability, p, for r such that a type $v \in [v_n, v_{n+1}]$ will weakly prefer r to his equilibrium report r_n is given by

$$1 - (M+1)p = [1 - (M+1)p_n]\frac{v - r_n}{v - r}.$$

Because $r > r_n$, the right hand side is decreasing in v, so the $v \in [v_n, v_{n+1}]$ with the largest such p is the upper bound, v_{n+1}. A parallel argument establishes that the $v \in [v_{n+1}, v_{n+2}]$ who has the largest such p is the lower bound, v_{n+1}. This shows that among types in the two adjacent pools who make reports just above and just below the out-of-equilibrium report, D1 requires that the auditor believe it is only the boundary type who could have sent the report. A similar argument also shows that the auditor will believe it is this boundary type among all other pools.

We now ask what these beliefs imply about the auditor's choice of audit probabilitites for out-of- equilibrium reports. Consider first the out-of-equilibrium reports $r \in (0, v_1 - c)$ that are common to every equilibrium of Proposition 2, including the maximally informative equilibrium. The auditor must believe all these are sent only by v_1. But since $v_1 - r > c$, he will audit all these with probability one. This is fine, since the manager will then be deterred from sending these reports, as was desired.

Now consider any other out-of-equilibrium report $r \in (r_n, r_{n+1})$, bounded by two pools. By Proposition 2, to convince the auditor to audit r_{n+1} with the required probability we must have

$$r_{n+1} = E(v \mid v \in [v_{n+1}, v_{n+2}]) - c.$$

In particular, $r_{n+1} > v_{n+1} - c$, so there is an out-of-equilibrium report $r \in (r_n, r_{n+1})$, but sufficiently close to r_{n+1}, such that $r > v_{n+1} - c$ or $c > v_{n+1} - r$. Therefore, the auditor will not audit this r, and since it is not audited, types who should have chosen higher reports will now choose here, thus destroying every equilibrium containing a higher pool.

The maximally informative equilibrium also contains no reporting at the highest reports, $r \in [V - c, V]$. D1 requires that the auditor believe type V sent these reports and so he will not audit, as in the Lemma, which is as specified for the maximally informative equilibrium.

Turning finally to the commitment-like equilibria of Proposition 3, note that reports in $(r^* - c, r^*]$ must never be chosen by the manager. If one were, then $v = r^*$ is the largest that the firm value could be, because these reports are being audited with probability $1/(M + 1)$ and values $v \in (r^*, V]$ would prefer to choose r^* and not be audited. Consequently, the auditor would not audit such a report, a contradiction, and so reports in $(r^* - c, r^*]$ are out-of-equilibrium. By similar arguments as above, D1 requires that the auditor believe that $v = r^*$ sent such report, r; so $v - r < c$ and the auditor will not audit r, thus eliminating these equilibria as well.

Proof of Proposition 5:

The $p_i(r)$ have been defined so that the probability of discovery is $p(r)$. Since the manager cares only about this probability of discovery, he will be willing to report $r(v)$. Conversely, suppose that the manager reports according to $r(v)$. Then $p(r)$ solves

$$\begin{aligned} \max \quad & E\left[v - r + p(c' - (v - r)) \mid r\right] \\ = & E\left[v - r \mid r\right] + p[c - \pi E(v - r \mid r)]/\pi \end{aligned}$$

so, $p[c - \pi E(v - r \mid r)]$ is zero for every r. Therefore, whenever, $p(r) > 0$ in perfect auditing, the manager will be willing to choose $p_i(r) > 0$ at any stage in imperfect auditing, and conversely as well. In particular, the manager will be willing to choose a combination of $p_i(r)$ such that

$$p(r) = 1 - \prod_{i=0}^{\infty}(1 - p_i(r)\pi)$$

as is required.